⟨S|P|S⟩ swiss physical society

Nr. 60
Januar 2020

SPG MITTEILUNGEN
COMMUNICATIONS DE LA SSP

AUSZUG - EXTRAIT

History and Philosophy of Physics (26)

On Lemaître's inhomogeneous cosmological model of 1933 and its recent revival

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1 Introduction

The previous two talks at this symposium by Harry Nussbaumer and Jean-Pierre Luminet have clearly shown the crucial role of Georges Lemaître in the development of what we now call the standard model of cosmology. Already in 1933 he developed a more general model by relaxing the high symmetries of the Friedmann-Lemaître (FL) models, which are spatially homogeneous and isotropic. His extended model is inhomogeneous, but he kept the isotropy. Otherwise things would have become at the time hopelessly difficult. Only now – with the power of computer systems – it has become possible to study the evolution of inhomogeneous and anisotropic models in the framework of GR.

Lemaître's inhomogeneous model experienced an astonishing revival after 2000 when observations convincingly showed that the expansion of the universe is accelerating since about 2 billion years ($z \simeq 0.6$). This fact was almost universally attributed to the presence of a mysterious form of so-called *dark energy*, for instance a positive cosmological constant (still compatible with all observations).

Since the required *magnitude* of dark energy is a mystery, a minority of cosmologists has afterwards investigated the possibility that the observational findings might be caused by inhomogeneities in the distribution of matter and other quantities. More concretely, it was suggested that we live in an underdense region of the universe centered not far from us, and do not need dark energy.

I thought it would be fitting for this meeting on the occasion of Lemaître's 125th birthday to present his inhomogeneous model and to summarize its recent applications in cosmology ([1], English translation [2]).

2 Lemaître shows that the "Schwarzschild singularity" is apparent

I begin with a late Sect. 11 of this paper, which deviates from the central theme, entitled: **Schwarzschild's Exterior Field**. Almost all researchers in the field have overlooked this most remarkable contribution. (I learned about it only quite recently.)

Lemaître shows in this section that the so-called Schwarzschild singularity, that disturbed relativists over decades, is spurious. In his own words:

> "***The singularity of the Schwarzschild field is thus a fictitious singularity***" ([2], p. 676, emphasis added).

He showed this by transforming the standard form of the metric to new coordinates which are defined by a congruence of freely falling test particles in radial directions, starting at rest infinitely far away (radial parabolic motion) [1]. If Lemaître's insight would have been generally appreciated, the history of black hole physics would have been different, because the apparently singular Schwarzschild surface would probably soon have been understood as an event horizon [8].

[1] In 1963 Novikov introduced coordinates adapted to a congruence of non-parabolic initial conditions. Their relation to the standard Schwarzschild coordintes is much more complicated.

3 Qualitative description of Lemaître's inhomogeneous model

What Lemaître mainly does in the paper is to derive from Einstein's field equations the basic equations describing the time-dependent behavior of a spherically symmetric dust of stars or related models of the universe (see Fig. 1). For a long time his work was known under the name "Tolman model" (sometimes "Tolman-Bondi" model). This is strange since Tolman made no secret of the fact that the work was Lemaître's and quoted Lemaître. However, Bondi did not cite Lemaître. I could refer to papers, for instance in the *Astrophysical Journal* as late as 1995 by well-known authors, who used the "Tolman-Bondi" model for testing a new code. J. Peebles was one of the few who always cited the works by Lemaître in his books on cosmology. Most authors now use the term "Lemaître-Tolman-Bondi (LTB) model".

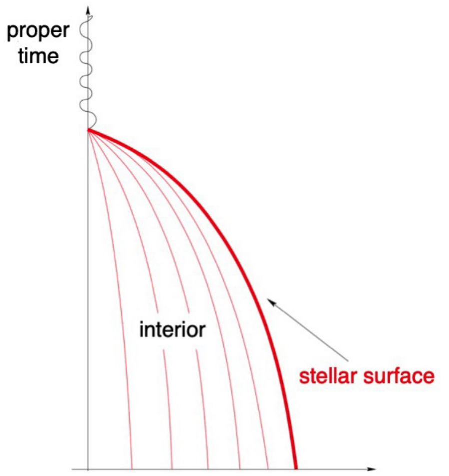

Figure 1: Worldlines of dust shells in spherical collapse. They all reach the singularity at the same proper time.

The famous work by J. Robert Oppenheimer and his student Hartland Snyder in 1939 on the gravitational collapse of a simplified stellar model [9] is a special application of Lemaître's equations, which they did not know. In their paper they describe crucial aspects of the collapse to a black hole, but because of the apparent Schwarzschild singularity a full understanding came only many years later. For the problem of a smooth matching of the internal and the external metric of the collapsing star, the new regularising coordinates of Lemaître for the Schwarzschild geometry would also have been significant.

The straightforward extension of Lemaître's treatment of the problem to ideal fluids with non-vanishing pressure was developed in an important paper by Misner and Sharp in 1964 [10]. Their equations became an essential part for numerical simulations of spherically symmetric collapse calculations and supernova explosions in the 1980s. More realistic collapse calculations, not assuming spherical symmetry, became possible with the impressive developments of numerical relativity and by a continued increase of computer power. This has become a vast field.

The technical description of the LTB model is postponed (s. Appendix), and we turn to observational tests.

4 Recent confrontation with observations and results of the LTB model

The observations of luminosities of supernovae of type Ia shortly before 2000 were almost universally interpreted as the result of an accelerated expansion of the universe, caused by *dark energy* (for instance a positive cosmological constant). Additional independent observations, in particular the measurements of the anisotropies in the cosmic microwave background and galaxy surveys, supported this interpretation with increasing accuracy.

Since the required *magnitude* of dark energy is a mystery, a minority of cosmologists in recent years has investigated the possibility that the observational findings might be caused by inhomogeneities in the distribution of matter and other quantities. More concretely, it was suggested that we live in an under-dense region of the universe centered not far from us, and do not need dark energy. Since the cosmic microwave background is highly isotropic, it is reasonable to start with simplified spherically symmetric but *inhomogeneous* models. For this reason, the LTB model was revived. (This can be regarded as an example for nonlinear deviations of FL-models. With the tools of numerical relativity more general studies are possible.)

The model is determined by the matter density $\rho(t_0,r)$ at the present time (apart from a few cosmological parameters) that is constrained by observational data. In what follows I concentrate on the detailed analysis in [11] (that contains references to related work). The authors assume that the early Universe was homogeneous until the time of recombination and followed standard physics, including decoupling. They use the measured local Hubble rate $H_0 \simeq 74$ km s^{-1} Mpc^{-1}, the supernovae data and the angular diameter distance to the surface of last scattering, D_A = 12.80 ± 0.068 Mpc from the *Planck* data. For the latter they follow a method for analyzing the CMB anisotropies in a manner that is as independent as possible of late-time cosmology [13], without using perturbation theory on an LTB background (we postpone explanations on this important fact). The following main conclusions result:

(i) The local Hubble rate and supernovae data can easily be fitted (for $\Lambda = 0$); the data favor the formation of large and deep voids (see Fig. 2).

(ii) "Model-independent" constraints from *Planck* + supernovae data imply an ***unrealistically low value of the local Hubble rate***, $H_0 \approx 39$ km s^{-1} Mpc^{-1}.

As [11], p. 8, put it, "*LTB models with a constant bang time function and zero cosmological constant are inconsistent with current data*" (emphasis added).

After this, the final sections of [11] are devoted to LTB models with a non-vanishing cosmological constant. It should be recalled that based on the famous Lovelock theorem any

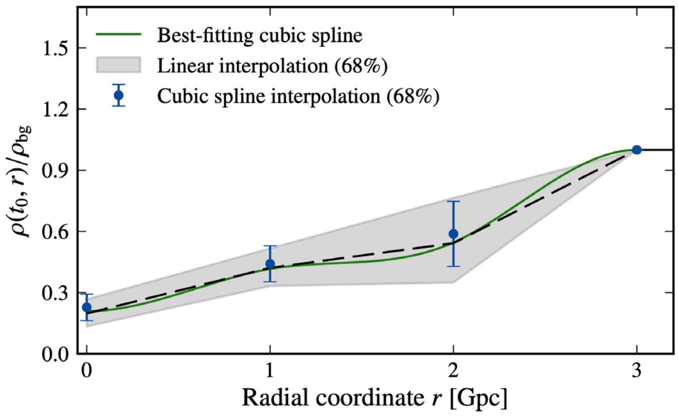

Figure 2: Deviations of $\rho(t_0, r)$ from spacial homogeneity for $r < 3$ Gpc for $\Lambda = 0$. The models were forced to converge to the background density at $r = 3$ Gpc (Fig. 3 from [11]).

metric theory of gravity has, without convincing counterarguments, at least two coupling constants: Newton's constant G and the cosmological constant. This was also Lemaître's view. He repeated this very clearly in his contribution to the famous Schilpp volume "Albert Einstein: Philosopher - Scientist" [16], entitled: *The Cosmological Constant*. I quote from Section 1:

> *Even if the introduction of the cosmological constant "has its sole original justification, that of leading to a natural solution of the cosmological problem" (Einstein), it remains true that Einstein has shown that the structure of his equations quite naturally allows for the presence of a second constant beside the gravitational one. This raises a problem and opens possibilities which deserve careful discussion. The history of science provides many instances of discoveries which have been made for reasons which are no longer considered satisfactory. It may be that the discovery of the cosmological constant is such a case.*

This is what cosmological data told us.

The authors of [11] arrived with their detailed analysis at the result that the data is, of course, better fitted under the LTB model than under the FL model (with Λ), but that the improvement is "almost negligible". So *current data show that LTB models on Gpc-scales must be close to FL models with* $\Lambda \neq 0$ (see Fig. 3).

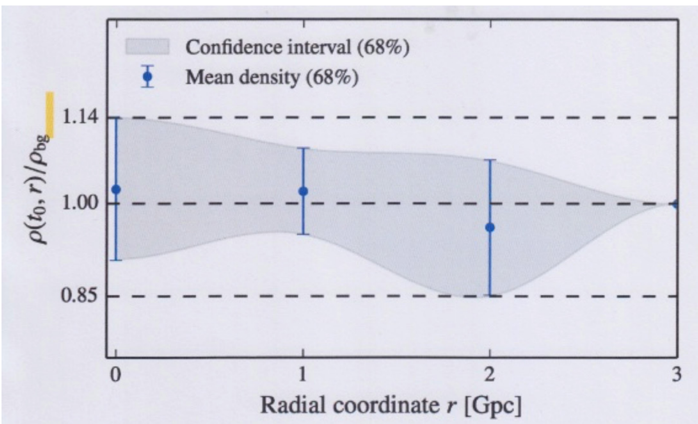

Figure 3: Deviations of $\rho(t_0, r)$ from spacial homogeneity for $r < 3$ Gpc for $\Lambda \neq 0$. The models were forced to converge to the background density at $r = 3$ Gpc (Fig. 10 from [11]).

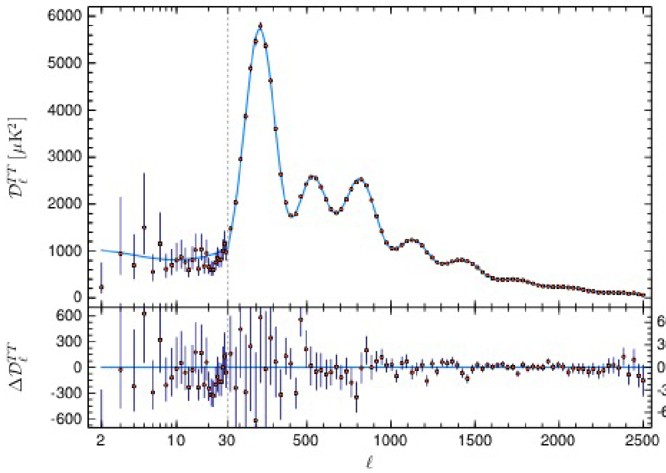

Figure 4: Planck temperature power spectrum as measured by the Plack satellite and fit of an FL model. Figure 1 from [12].

Their analysis of the *Planck* data (Fig. 4) is, as they admit, limited since linear perturbation theory for an LTB background is very complicated and still not sufficiently developed. Because this background has less symmetries than a FL background, gauge-invariant perturbations in a spherical harmonic decomposition are coupled by systems of linear *partial* differential equations, and not by hierarchies of *ordinary* differential equations as for the simpler FL models. (This is familiar from other fields of physics.) For existing attempts we refer to [17].

Without such analysis certain parameters are, however, well determined by the data. We illustrate this for the angular diameter distance D_A to the last scattering surface. This can be obtained from the CMB observations, with little dependence on the cosmic evolution from this surface to the present, in particular if inhomogeneities affect the evolution. Qualitatively, this can be understood as follows.

The formation of the radiation fluctuations – the primordial spectra – are in our standard model very well understood and worked out with high precision. While the observed spectra, for instance the temperature-temperature correlation in Fig. 3, are influenced by the later evolution, the sequence of peaks and troughs reflects a characteristic scale on the surface of the last scattering. A priori, it is, however, possible that we see a scale that was originally formed during the early universe by standard cosmology, but was afterward changed in an unknown manner by post-recombination evolution.

Physically, the characteristic scale imprinted on the last scattering surface (LSS) is the so-called *sound horizon* at decoupling, which is the distance sound waves in the photon-baryon plasma can travel from early times until they reach this surface. Vonlanten, Räsänen & Durrer [13], and a later extension by Audren [14] using the *Planck* results, showed that the late-time cosmology preserves this scale for short distance fluctuations. More precisely, they demonstrated that the part of the spectrum with l larger than about 40 is predominantly rigidly shifted ($l \to S^{-1}l$) and its height is re-scaled. Indeed, it turned out that with such a shift an excellent fit to the observed power spectra is achieved. The resulting scale parameter S determines directly the angular distance D_A to the last scattering surface: $D_A = 12.8^{+0.071}_{-0.065}$ Mpc. (For technical details, I refer to the cited papers.)

It may help to illustrate this for the class of FL models with its various cosmological parameters. The power spectra at the time of recombination do not depend on the parameters Ω_K and Ω_Λ, since they play no role in the evolution until that time. They modify, however, the post-recombination spectra. It turns out that the present power spectra for sufficiently large l values ($l \gtrsim 30$) do not fix their values individually, but a certain function of them, the so-called shift parameter R. This is closely related to S and the angular diameter distance D_A. So this distance is determined by the power spectrum at short distances. It is this aspect that is generalized in a model-independent way in the cited papers [13], [14]. Briefly, the observed power spectrum for ($l \gtrsim 30$) can be fitted by treating D_A as a free parameter, whose physical origin is left open. We have seen that for the LTB model it provides an important restriction of the initial density distribution.

I add to this discussion the following remark on local and global values of the Hubble parameter. In the framework of the FL models it was possible to deduce a precise value for this parameter from fits to the *Planck* power spectra of the cosmic background radiation: $H_0 = 67.8 \pm 0.9$ km s^{-1} Mpc^{-1}. This disagrees with the local value 74 (in the same units) that was obtained earlier by astronomers. This 3.6σ discrepancy caused in recent years lots of discussions and speculations. Last summer a new value for the local H_0 was published by W. Friedman and collaborators, based on a new independent method (using the tip of the red giant branch). With this, the situation is now confusing, since the new value lies almost exactly in the middle between the previous two: $H_0 = 69.8 \pm 0.8$ km s^{-1} Mpc^{-1}. This value agrees with the global *Planck* value at the 1.2σ level, and is 1.75σ below the previous local one. For a recent summary of the current measurements of H_0, using various methods including gravitational lensing, see [18].

Planned future observations will hopefully decide whether a discrepancy remains for the FL cosmology. Recent cosmological general relativistic simulations [19] strongly indicate that a 3.6σ discrepancy cannot be explained with inhomogeneities.

Appendix. Basic equations of the LTB model

For readers familiar with the essentials of General Relativity we present in this final section the basic equations of the LTB model. For a spherically symmetric dust model about a distinct central worldline the metric has in suitable coordinates (t, r, ϑ, φ) the form (see [15], Sect. 4.10.1)

$$g = -dt^2 + e^{2b(t,r)}dr^2 + R^2(t,r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2). \qquad (1)$$

Einstein's field equations imply that

$$e^{2b(t,r)} = \frac{(R'(t,r))^2}{1-k(r)}, \qquad (2)$$

where $k(r)$ is a function of r alone, for which the boundary condition $k(0) = 0$ can be imposed. (In what follows a prime denotes the partial derivative with respect to r and a dot the time derivative.) Another arbitrary function is a kind of mass function, $M(r)$, defined by

$$M'(r) = 4\pi R^2(t,r)\,R'(t,r)\,\rho(t,r),\ M(0) = 0, \qquad (3)$$

where $\rho(t, r)$ is the matter density (of dust, for instance dark matter). Einstein's field equations imply that M is time independent.

The dynamical field $R(t, r)$ of the metric satisfies the "Hamiltonian constraint" equation

$$\frac{\dot{R}^2}{R^2} = \frac{2GM(r)}{R^3} + \frac{8\pi G}{3}\rho_\Lambda - \frac{k(r)}{R^2}, \qquad (4)$$

which is a differential equation for $R(t, r)$ as a function of t with r as a parameter. On the right we have included, following Lemaître, the contribution of the Λ term ($\rho_\Lambda = \Lambda/8\pi G$).

The basic dynamical equation (4) generalizes the Friedmann equation. For the discussion of these basic equations we introduce several derived functions:
$H(t, r)$ denotes the local Hubble rate

$$H(t,r) := \frac{\dot{R}(t,r)}{R(t,r)}, \qquad (5)$$

and $H_0(r)$ its value at the present time t_0

$$H_0(r) := H(t_0, r). \qquad (6)$$

We define local density parameters $\Omega_M(r)$, $\Omega_\Lambda(r)$, $\Omega_K(r)$ by

$$2GM(r) =: H_0^2(r)\,\Omega_M(r)\,R_0^3(r),\ \Omega_\Lambda(r) := 8\pi G\frac{\rho_\Lambda}{H_0^2(r)},\ \Omega_K := 1 - \Omega_M - \Omega_\Lambda, \qquad (7)$$

where $R_0(r) := R(t_0, r)$. With these definitions we can rewrite the Hamiltonian constraint (6) as

$$H^2 = H_0^2\left[\Omega_M\left(\frac{R_0}{R}\right)^3 + \Omega_\Lambda + \Omega_K\left(\frac{R_0}{R}\right)^2\right]. \qquad (8)$$

The local spatial curvature $k(r)$ is given by

$$k(r) = -H_0^2\Omega_K R_0^2 = 1 + H_0^2(1 - \Omega_M - \Omega_\Lambda)\,R_0^2(r). \qquad (9)$$

Equation (8) looks like the well-known equation in Friedmann-Lemaître (FL) models [2]

$$H^2 = H_0^2\left[\Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda + \Omega_K\left(\frac{a_0}{a}\right)^2\right], \qquad (10)$$

where $a(t)$ is the scale factor, but with r-dependent functions $H_0(r)$, $\Omega_M(r)$, etc. Lemaître did a lot of analytic work based on these equations, but solutions can be obtained only for idealized problems. This part is no longer really interesting, since we now have the help of computers. For instance, for the luminosity-redshift relation, one must solve a pair of nonlinear differential equations.

2 For these $R(t, r) = a(t)r$.

Since the radial coordinate r is arbitrary, one can eliminate one arbitrary function. For instance, by choosing the gauge $R(t_0, r) = r$, the mass function $M(r)$ is according to (5) determined by the density profile $\rho(t_0, r)$ at the present time t_0:

$$M(r) = 4\pi G\int_0^r \rho(t_0, r')\,r'^2\,dr'. \qquad (11)$$

From (11) we obtain for the present time

$$t_0 = \frac{1}{H_0(r)}\int_0^1 \frac{dx}{\sqrt{\Omega_M(r)\,x^{-1} + \Omega_\Lambda(r)\,x^2 + \Omega_K(r)}}. \qquad (12)$$

Here we left out a possible additive r-dependent term (the so-called bang time function), assuming that the Big Bang occurs synchronously, as in FL models. (This should, in any case, be much smaller than t_0.) Because we want to match the solution to a homogeneous FL solution in the outer space, t_0 is chosen to be equal to the FL value (for the best fit). With all this the previous equations *determine all quantities of interest by* $\rho(t_0, r)$ plus the Hubble constant and the cosmological constant.

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