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Planck and the Extraterrestrials or what the new SI means for astronomy

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In 1899, during his struggle to understand blackbody radiation but about a year before he found the key, Planck proposed a curious set of units [1]. In these natural units, as he called them, the unit of mass m is $\sqrt{hc/G}$ which comes to about 55 μg. The length unit is $h/(mc)$, the time unit is $h/(mc^2)$, while the temperature unit is mc^2/k. (Reading the original today, one needs to substitute $f \to G$, $b \to h$, $a \to h/k$.) Planck then declared that these units would be recognized by all cultures as fundamental, even extraterrestrial or non-human ones ("ausserirdische und aussermenschliche Culturen").

Planck was not the only one to propose a system of units based on fundamental constants [2], but he was the most prescient. In Planck's system there is no prototype mass, length, time, or temperature. Instead there are dimensional coefficients that appear in multiple equations of physics, which define the units implicitly. In the 21st century the International System of units concluded that implicit definition of units is not a bug, it's a feature. In 2018 the SI decided that only the second shall be defined by a particular physical process (a spectral line in Cs). All other units are defined implicitly, through defined values of physical constants, including (but of course) Planck's constant. Any applicable equation of physics can be used for the realization of units [3].

The reforms in the SI were not especially designed to communicate with extraterrestrials. The decision to adopt implicit definitions was pragmatic, in that explicit definitions were sometimes overtaken by technology. In particular, the kilogram is best realized as a derived electrical unit, with mass measured by way of the quantum Hall effect and the Josephson effect. That would be sternly frowned upon by the old SI, but the new SI is cheerfully agnostic about it. The new SI also removes conflicts with some common scientific usages. For example, the electronvolt is not an SI unit. However, if we understand mass in electronvolts as a shorthand for the value of mass times c^2/e in volts, there is no conflict with the new SI.

Planckian units can be useful useful for theoretical work, provided one changes to standard units for comparing with observable quantities [4]. Meanwhile, the community of scientists engaged with extraterrestrial things have steadfastly resisted adopting SI units. Even the proponents of SI units among astronomers [5] have not advocated a complete change to the SI. Why is that? The answer may lie to some extent in sociological factors, but there has been a scientific reason. The key is the constant that Planck included but the new SI does not: G. The gravitational constant is uniquely weak and uniquely hard to measure, and even now is known to only four significant digits [6]. The so-called solar mass parameter $GM_\odot$ is known to eight significant digits in Newtonian gravity, two more with general relativity. But if you split that product into G and $M_\odot$, in order to write the mass of the Sun in kilograms, your value of the gravitational field will be only good to four significant digits. You cannot navigate a spacecraft on four significant digits. End of story for SI in astronomy. Or so it used to be.

After the reforms, however, the SI no longer requires you to express the solar mass in kg — but we'll get to that. We remark first that distances in light-seconds are compliant with the new SI. Like the electronvolt, a light-second can be understood as not itself a unit, but an abbreviated way of referring to the equivalent light travel time. On the other hand, which astronomer would wish to lose the cultural legacies of the astronomical unit of length (au), and the parsec? Fortunately, metrology and cultural legacies can be reconciled. To see how, we need only to see how modern German has kept the old unit of a Pfund, but has rounded its meaning to exactly 500 g. The au and parsec have serendipitously round values: 1 au $\simeq c \times 500$ s to better than 1 %, while a parsec is just 3 % over $c \times 10^8$ s. Many astronomical applications do not require 1 % precision, and there "rounded" au and parsecs would work just fine.

In the astronomy literature one finds many further units (Ångström, Gauss, Jansky) that are simply powers of 10 times SI units, and need not detain us here. There are also parody units such as gallons Mpc^{-1} for cross-sections (it depends on the type of gallon, but is about a kilobarn), which would need an article to themselves. The only remaining unit that needs discussing is the optical magnitude scale. Optical magnitudes are a measure of brightness going back to classical times, but like other units they have been progressively redefined, and are now considered SI-traceable. A zero-magnitude source (such as the star Vega) corresponds to a flux of roughly 10^{10} photons m^{-2} s in a typical spectral band. More precisely, zero magnitude is 5.480×10^{10} photons m^{-2} s per logarithmic spectral interval, with each 5 mag being 100 times fainter. Since modern optical cameras are invariably counters of photons, it makes sense to simply use photon fluxes rather than optical magnitudes, with "rounded" magnitudes used for historical comprehension.

Now we come to the kilogram, or avoiding the kilogram when necessary. Spacecraft dynamics has long used $GM_\odot$ in m^3 s^{-2}. In pulsar timing, $GM_\odot/c^3$ in seconds is usual. In the new SI, by analogy with electronvolts, we can meaningfully measure mass as GM/c^3 in "gravity-seconds". The solar mass is about 5 micro-(gravity)-seconds. This does not mean that kilograms are to be avoided in astrophysics. Rather, gravity-seconds can be useful where kilograms would needlessly propagate the uncertainty in G, which is typically the case for orbital processes. Together with distances in light-seconds and dimensionless velocities, gravity-seconds allow for some elegant simplifications in the description of all kinds of orbital processes, classical and relativistic [7]. Some constructions arise that seem very strange at first: in particular density in gravity-seconds per cubic light-seconds, or power in gravity-seconds per second. But these are not really unphysical at all. In gravitational phenomena, a density ρ is always associated with a time scale $(G\rho)^{-1/2}$, so density as frequency squared does make sense. As for dimensionless power in gravity-seconds per second, this is just power in units of the well-known Planck power c^5/G, which is the luminosity scale in gravitational waves for merging black holes, irrespective of their mass.

To illustrate working with the new SI in astronomy, let us consider an example that sounds like science fiction, but is in fact a serious design study: the solar gravity lens mission [8]. It envisions a telescope larger than the solar system with the sun's gravitational field as its main lens, for mapping the surfaces of extra-solar planets. The essential physical effect is that light rays going past the sun with impact parameter R are gravitationally deflected by an angle $4GM_\odot/(c^2R)$. In particular, the deflection of light from a star that is barely covered by the sun will make that star appear to be just outside the rim of the sun. The deflection at the rim of the sun is well known to be 1.75 arcsec and its first measurement a century ago is the stuff of legend [9]. Knowing the solar mass in gravity-seconds and approximating the solar radius as $R_\odot \approx c \times 2$ s, we can trivially work out the gravitational deflection angle $\alpha = 4GM_\odot/(c^2R_\odot) \approx 10^{-5}$ radians, which is indeed a little under 2 arcsec. If we go far enough from the sun that its apparent radius on the sky becomes smaller than the deflection angle, a remarkable phenomenon will present itself: light sources precisely behind the sun will appear as luminous rings (known as Einstein rings) around the sun. The threshold distance to see an Einstein ring is $R_\odot/\alpha \approx c \times 2 \times 10^5$ s or ≈ 400 au. (Actually about 550 au with less drastic approximations.) The advantage of an Einstein ring is that it enormously magnifies the source, so much so that a 1 m telescope located at the right place in deep space could resolve features down to 10 km on an exoplanet at 30 parsecs. That level of detail is comparable to what the Hubble Space Telescope can see on Mars. The image-reconstruction problem involved would be a major challenge in its own right, because the magnification produced by Einstein rings is highly anisotropic, but simulations indicate that it is solvable and that images could indeed be obtained. The task that remains, then, is to fly a fleet of space telescopes to appropriate locations beyond 500 au. And within one lifetime, if possible, please. The currently most promising strategy for doing this turns out to be (who would have guessed?) solar sails. Solar sails also provide a nice example of how kilograms and gravity-seconds can co-exist. Radiation pressure from the sun is, like gravity, inverse squared in the distance. Hence the acceleration on a spacecraft with a solar sail will take the form

$$\frac{SF_\odot}{r^2} - \frac{GM_\odot}{r^2}$$

where r is the distance from the sun, $F_\odot/r^2$ is the radiation pressure from the sun, and S is the effective sail area per unit mass. For simplicity we are neglecting the gravity of the planets and further non-gravitational perturbations. The solar-sail term in effect reduces the gravitating mass of the sun, and could even reverse its sign if S is large enough. Let us consider both terms at $r = r_\oplus = 1$ au. Knowing the solar mass in gravity-seconds we easily get $GM_\odot/r_\oplus^2 \approx 6$ mm s^{-2}. To estimate $F_\odot/r_\oplus^2$ a convenient way is to recall the solar constant, which is the energy flux from the Sun at the top of the Earth's atmosphere. This energy flux is, of course, in photons, and the energy of a photon is c times its momentum. Holding up a mirror normal to sunlight reflects the photons and imparts twice their momentum to the mirror. Hence, multiplying the solar constant by $2/c$ gives the pressure on an optimally-aligned solar sail 1 au from the sun. The value of the solar constant is 1361 W m^{-2} with small variations which we will disregard here, and the corresponding pressure will be a little under 10^{-5} Pa. Comparing the terms, we see that we would need $S \approx 600$ m^2 kg^{-1} to make the sun a net repeller. This may or may not be technologically feasible, but it is actually not necessary, thanks to a further cunning plan. This is to first plunge the spacecraft in a very eccentric orbit close to the sun, and then at perihelion deploy the solar sail. Without the solar sail, the spacecraft would simply have returned to repeat the orbit, but now it can sail out of the solar system on its mission to the focal region of the solar gravity lens.

Even with such a daring space mission, it will probably be some time before we make contact with non-human or extraterrestrial cultures, so that we can ask them if they indeed share Planck's views on units. Yet in the 120 years since Planck drew special attention to physical constants, they have turned out to have meanings not even Planck in 1899 had imagined. As we all know, the following year Planck himself discovered that h (formerly known as b) was about quantization. A few years later the formal relation between mass and energy through c^2 turned out to be real. Starting in the 1920s $(hc/G)^{3/2} \times$ (proton mass)$^{-2}$ was discovered to be the mass scale of stars. Later in the 20th century, $2e/h$ and h/e^2 turned out to be macroscopically observable as Josephson's constant and von Klitzing's constant. And just in the last few years, c^5/G has been observed to be the power-output scale for merging black holes. We can only wonder what more surprises await us.

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